From the graph, we see the sequence \((-1)^n \frac{n+1}{n}\) is divergent, since it oscillates between 1 and -1 (approximately).

\[
\lim_{n \to \infty} \frac{2n}{2n+1} = \lim_{n \to \infty} \frac{2}{2+\frac{1}{n}} = 1 \text{ so } \lim_{n \to \infty} \arctan\left(\frac{2n}{2n+1}\right) = \arctan(1) = \frac{1}{4} \pi.
\]
From the graph, it appears that the sequence converges (slowly) to $0$. 

$$0 \leq \frac{|\sin n|}{\sqrt{n}} \leq \frac{1}{\sqrt{n}} \to 0 \text{ as } n \to \infty,$$

so by the Squeeze Theorem and Theorem 6, \( \left\{ \frac{\sin n}{\sqrt{n}} \right\} \) converges to zero.

#46

From the graph, it appears that the sequence is converging to $5$. 

$$5 = \sqrt{5^n} \leq \sqrt{3^n + 5^n} \leq \sqrt{5^n + 5^n} = \sqrt{2} \sqrt{5^n} = 5 \sqrt{2} \to 5 \text{ as } n \to \infty.$$  

#47 Note: \[
\frac{1 \times 3 \times 5 \cdots (2n-1)}{(2n)^n} = \frac{(2n)!}{(2n)^n n! 2^n}
\]

From the graph it appears that the sequence approaches $0$. 

$$0 < a_n = \frac{1 \times 3 \times 5 \cdots (2n-1)}{(2n)^n} = \frac{1}{2n} \times \frac{3}{2n} \times \frac{5}{2n} \cdots \frac{2n-1}{2n} \leq \frac{1}{2n} \times 1 \times 1 \cdots 1 = \frac{1}{2n} \to 0 \text{ as } n \to \infty.$$  

So by the Squeeze Theorem \( \left\{ \frac{1 \times 3 \times 5 \cdots (2n-1)}{(2n)^n} \right\} \) converges to $0$. 

2
#49 (a) $a_n = 1000(1.06)^n \Rightarrow a_1 = 1060, a_2 = 1123.60, a_3 = 1191.02, a_4 = 1262.48,$
$a_5 = 1338.23$
(b) $\lim_{n\to\infty}a_n = 1000\lim_{n\to\infty}(1.06)^n$, so the sequence diverges
($\lim_{n\to\infty}r^n$ only converges if $-1 < r \leq 1$)

#50 The first forty terms are:
11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4
When $a_1 = 25$ the first forty terms are:
25, 76, 38, 19, 58, 29, 88, 44, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4
The famous Collatz CONJECTURE is that this sequence always reaches 1, regardless of the starting point $a_1$.

#51 Since $|nr^n| = |r^n| \geq |r^n|$, if $|r| > 1$ then $\{nr^n\}$ diverges. If $|r| < 1$ then:
\[ \lim_{x\to\infty}nx^n = \lim_{x\to\infty}x^n \left( \frac{L^H}{r^n} \right) = \lim_{x\to\infty} \frac{\left( \frac{L}{r^n} \right)}{x} = 0, \]
so $\{nr^n\}$ converges to 0 for $|r| < 1$.
If $r = 1$, $nr^n = n$, and $\{n\}$ is divergent. Finally if $r = -1$, $nr^n = (-1)^n n$, and $\{(-1)^n n\}$ is divergent.

#55 $a_n = \frac{1}{2n+3}$ is decreasing since $a_{n+1} = \frac{1}{2(n+1)+3} = \frac{1}{2n+5} < \frac{1}{2n+3} = a_n$ for each $n \geq 1$.
The sequence is bounded since $0 < a_n \leq \frac{1}{3}$ for all $n \geq 1$. (Note that $a_1 = \frac{1}{3}$)

#57 $a_n = \cos \left( \frac{\pi n}{2} \right)$ is not monotonic. The first few terms are: 0, -1, 0, 1, 0, -1, 0, 1, ...
In fact, the sequence consists of the terms 0, -1, 0, 1 repeated over and over again in that order.
The sequence is bounded since $|a_n| \leq 1$ for all $n \geq 1$.

#59 $a_n = \frac{n}{n^2+1}$ is decreasing since for $f(x) = \frac{x}{x^2+1}$ we have: $f'(x) = \frac{1-x^2}{(x^2+1)^2} \leq 0$ for $x \geq 1$. The sequence is bounded since $0 < a_n \leq \frac{1}{2}$ for all $n \geq 1$.

#61 $a_1 = 2^{\frac{1}{4}}$, $a_2 = 2^{\frac{1}{2}}$, $a_3 = 2^{\frac{3}{4}}$, ..., $a_n = 2^{\frac{2n-1}{2^n}} = 2^{1-(\frac{n}{2^n})} - 2$.

Page 756:
#15 $\sum_{n=1}^{\infty} 5 \left( \frac{2}{3} \right)^{n-1}$ is a geometric series with first term $a = 5$ and common ratio $r = \frac{2}{3}$.
Since $\left| \frac{2}{3} \right| < 1$, the series converges to $\frac{a}{1-r} = \frac{5}{1-\frac{2}{3}} = 15$.

#17 $\sum_{n=1}^{\infty} \left( \frac{2}{3} \right)^{n-1} = \frac{1}{3} \sum_{n=1}^{\infty} \left( \frac{2}{3} \right)^{n-1}$ the latter is a geometric series with first term $a = 5$ and common ratio $r = \frac{2}{3}$. Since $|r| < 1$ the series converges to $\frac{a}{1-r} = \frac{1}{1-(\frac{2}{3})} = \frac{3}{1} = 4.7$.
Thus the original series (which is also geometric) converges to $\frac{1}{4} * \frac{3}{4} = \frac{1}{4}.$

#19 $\sum_{n=1}^{\infty} \frac{2^{n}}{3^{n+1}} = \frac{1}{3} \sum_{n=1}^{\infty} \frac{2^{n}}{3^{n+1}} = \frac{1}{3} \sum_{n=1}^{\infty} \left( \frac{2}{3} \right)^{n}$ is a geometric series with common ratio $\frac{2}{3}$. 

3
Since \( \left| \frac{1}{n} \right| \leq 1 \) the series diverges.

#21 Diverges by the divergence test 
\[
\lim_{n \to \infty} \frac{n}{n^3} = \lim_{n \to \infty} \frac{1}{(1 + \frac{1}{n})} = 1 \neq 0.
\]

#23 Using partial fractions, the partial sums are:
\[
s_n = \sum_{i=2}^{n} \frac{2}{i^2 - 1} = \sum_{i=2}^{n} \left( \frac{1}{i - 1} - \frac{1}{i + 1} \right) = (1 - \frac{1}{2}) + (\frac{1}{2} - \frac{1}{3}) + (\frac{1}{3} - \frac{1}{4}) + \cdots + (\frac{1}{n-1} - \frac{1}{n}) + (\frac{1}{n} - \frac{1}{n+1})
\]
This sum is a telescoping and 
\[
s_n = 1 + \frac{1}{2} - \frac{1}{n-1} - \frac{1}{n} \to \frac{3}{2} \text{ as } n \to \infty.
\]
Hence \( \sum_{n=2}^{\infty} \frac{2}{n^2 - 1} = \frac{3}{2} \).

#25 Diverges by the divergence test 
\[
\lim_{k \to \infty} \frac{k^2}{k^2 - 1} = \lim_{k \to \infty} \frac{1}{1 - \frac{1}{k^2}} = 1 \neq 0.
\]

#27 
\[
\sum_{n=1}^{\infty} \frac{3^n + 2^n}{6^n} = \sum_{n=1}^{\infty} \frac{3^n}{6^n} + \sum_{n=1}^{\infty} \frac{2^n}{6^n} = \sum_{n=1}^{\infty} \left( \frac{1}{2} \right)^n + \sum_{n=1}^{\infty} \left( \frac{1}{3} \right)^n = \frac{\frac{1}{2}}{1 - \frac{1}{2}} + \frac{\frac{1}{3}}{1 - \frac{1}{3}} = \frac{3}{2}.
\]

#29 
\[
\lim_{n \to \infty} a_n = \lim_{n \to \infty} \sqrt[n]{2} = \lim_{n \to \infty} 2^{\frac{1}{n}} = 2^0 = 1 \neq 0. \text{ So the series diverges by the divergence test.}
\]