Hmk #6 Central Limit Theorem Examples:

#1 Fish in a given lake have mean length of 16.7 inches with a standard deviation of 1.4 inches find the following probabilities.
(A) The probability that the mean length of 36 fish chosen at random is bigger than 17 ins.
(B) The probability that the mean length of 49 fish chosen at random is bigger than 17 ins.

Solution:
(A) \( \text{normalcdf}(17, E99, 16.7, 1.4/\sqrt{36}) = 0.0992714622 \) (about 9.9%)
(B) \( \text{normalcdf}(17, E99, 16.7, 1.4/\sqrt{49}) = 0.0668072287 \) (about 6.7%)

#2. A bottling company uses a filling machine to fill plastic bottles with a popular cola. In fact, suppose \( \mu = 350 \) ml and standard deviation \( \sigma = 4 \) ml.
(a) Take a random sample of 48 bottles. What are the mean and standard deviation of the sample mean contents \( x \) of these 48 bottles?
(b) What is the probability that the sample mean contents of the 48 bottles is less than 349 ml?

Solution:
(a) mean is 350, standard deviation is \( \frac{4}{\sqrt{48}} = .57735 \)
(b) \( \text{normalcdf}(-E99, 349, 350, \frac{4}{\sqrt{48}}) = 0.0416322192 \) (about 4.2%)

#3 The length of human pregnancies from conception to birth varies according to a distribution that has a mean of 264 days and standard deviation 16 days. Consider 36 pregnant women from a rural area. Assume they are equivalent to a random sample from all women.
(A) What’s the probability the sample mean length of pregnancy lasts less than 259 days?
(B) What’s the probability the sample mean length of pregnancy lasts more than 270 days?

Solution:
(a) \( \text{normalcdf}(-E99, 259, 264, \frac{16}{\sqrt{36}}) = 0.0303962972 \) (about 3%)
(b) \( \text{normalcdf}(270, E99, 264, \frac{16}{\sqrt{36}}) = 0.0122244334 \) (about 1.2%)

#4 A lightbulb manufacturer claims that the lifespan of its lightbulbs has a mean of 55 months and a st. deviation of 5 months. Your consumer advocacy group tests 64 of them. Assuming the manufacturer’s claims are true, what is the probability that it finds a mean lifetime of less than 54 months?

Solution:
\( \text{normalcdf}(-E99, 54, 55, \frac{5}{\sqrt{64}}) = 0.0547992894 \) (about 5.5%)

#5. According to a 1995 study, the mean family income in the US was $38,000 with a standard deviation of 21,000. If a consulting agency surveys 625 families at random, what is the probability that it finds a mean family income of more than $39,500?

Solution:
\( \text{normalcdf}(39500, E99, 38000, \frac{21000}{\sqrt{625}}) = 0.370727146 \) (about 3.7%)

#6 The mean serum cholesterol of a large population of overweight adults is 225 mg/dl and the standard deviation is 15 mg/dl. If a sample of 81 adults is selected at random, find the probability that the mean will be between 225 and 227 mg/dl.

Solution:
\( \text{normalcdf}(225, 227, 225, \frac{15}{\sqrt{81}}) = 0.3849302679 \) (about 38.5%)